Formulae as scientific stories

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ABSTRACT In science lessons many students struggle to apply the principles of rearranging formulae, even after coverage in maths. A structured approach is suggested that focuses on describing a narrative linking cause and effect before explicit mathematical terms are introduced.

Narratives

Once upon a time I got myself into bad habits with formulae, perhaps because I was so familiar with them. It can seem logical to start the lesson with the formula our students will see in the textbook, but they often do not see how apparently random letters can have meaning (Thompson, 2016). After all, in maths even the choice of letters is arbitrary, while in science they are shorthand for words or phrases.

If we take a step back from planning individual lessons, then it may help to consider a consistent approach. I model this by thinking of four steps students need to take if they are to succeed when using mathematical relationships. The balance between these will vary by topic and student, and at times I might return to an earlier stage when student work shows they need further guidance. In most cases, this is the sequence I would expect to go through with classes:

1. building understanding;
2. linking to other concepts, examples in life;
3. practising for fluency;
4. memorising for quick recall.

It is easy to forget, as a science specialist, that students will struggle with the mechanics of rearranging equations as well as the concepts themselves. Discussing this with maths colleagues is always worthwhile, and always difficult to find time for – quite apart from any departmental politics going on! For example, in maths lessons, students may be taught to use a 'balancing' method rather than the 'change side, change sign' approach with which we are often more familiar (Campton and Stevenson, 2014) (Box 1). Even confident science students may still need to include the explicit 'balancing' steps. As the result is the same, we may not realise that students are not rearranging the variables in quite the way we might expect. Providing commentary at each stage may help, but it still means a focus on mathematical procedures, not the scientific relationships.

BOX 1 Two approaches to solving equations

Balancing method			Change side, change sign method		
$3x + 4$	=	19	$3x + 4$	=	19
$3x + 4 - 4$	=	19 – 4	$3x$	=	19 – 4
$3x$	=	15	$3x$	=	15
$3x/3$	=	15/3	x	=	15/3
x	=	5	x	=	5

In practical work, science teachers make the distinction between activities being 'hands-on' and those that are also 'minds-on'. In the same way, we must ensure that when using maths in science lessons we provide real-world examples to which they can apply it. As well as experiencing, recording and analysing the mathematical relationship, we need to be sure they can *explain* it. The first two parts of this process, building understanding and providing context, are necessarily separate from the recall. I would argue that better understanding, especially in a form that lends itself to recall under pressure, makes the memorisation easier.

Telling scientific stories to build understanding

Some scientific quantities are derived; density and volume are properties of a situation or system that can be measured or worked out (Boohan, 2016: 93). Some properties are more abstract than others, and so harder for students to grasp. This is why teaching momentum at first as 'un-stoppableness' can be helpful! These relationships are mathematical, but they do not really demonstrate causation in a clear way. Others are different.

What makes science truly powerful is that it is *predictive*. Our subject is about causes and effects: *why* and *how*. So maybe we should be using that approach when we teach our students how quantities are related. You can think of this as a *narrative approach*, and it has the definite advantage that we can tell our students that we will leave the maths for later. By the time they realise they have been thinking mathematically – just not with numbers – the relationship is already established.

This approach uses the natural human tendency to explain the world using stories. For more information on this, I recommend the *Pan narrans* model that I first encountered in one of the *Science of Discworld* books (Pratchett, Stewart and Cohen, 2002). The suggestion is that stories are mental models that are communicated between individuals so understanding can be passed on. All teachers use this approach when stories are told to illustrate particular examples, whether as cautionary tales of experimental mistakes or parables of abstract ideas (Machacek, 2014). Humans are not the only tool-users, but arguably our greatest tools are the stories we tell to make sense of the world around us.

Although it might not look like one, *an equation is a story*:

- The **cause** is the starting point, where we begin. Even younger pupils should be able to tell you that this is the independent variable. Force and potential difference are good examples of common physical causes.
- The **conditions** of the situation might exaggerate what is happening. Some reduce it or dilute the effect. These are the factors that can make a difference: it is just that at the moment, they are kept fixed so they do not mess up the simple link being investigated. These are the control variables, with their chosen values. When applying forces to a trolley, we would usually keep mass the same. It is fairly intuitive that if the force acts on a larger mass, the effect will be smaller. In language students will access, it is being spread out more.
- The effect or **consequence** is the result of a change in our independent variable (the cause). Students are often told that this is the last column of the results table, and usually on the *y*-axis of a graph. When a resultant force acts on a mass, the result is acceleration. The three parts of the story allow students to build a mental model of how the quantities are related.
- The eternal classroom question is why should we care? The **context**, the moral of the story if you like, is about how the link between these quantities matters in the real world. This is also the time when we can point out where this story is embedded in other science topics, or other subjects in the curriculum. The students may have hobbies or interests where they have been relying on an imperfect or implicit understanding already. More formally, students should be able to explain examples they are given, or suggest a few of their own. If a car is loaded with more passengers, the same cause will lead to a smaller effect: a lower acceleration.

These four terms (in bold print above) can be applied to *any* scientific formula. The conditions include any constants, and there may be more than one control variable to consider. If students can be reminded to look for this structure then they have the narrative established. The details of mathematical operations may not be clear enough when using words, which means that they can see the need for the next step.

Telling mathematical stories

Using their science connectives, students can now explain what they would expect to happen if the cause or conditions are changed. This is science-specific vocabulary, just as much as words such as *electron* or *osmosis* (Ross, Fisher and Frey, 2009). They are moving towards quantitative explanations, using words such as *more* or *less*, *increase* or *decrease* (Box 2). This intermediate stage feels much less threatening than a set of numbers, and is not far from more specific descriptions such as '*the acceleration doubles*'.

And now we are doing maths, so we tell our students that it is time to be efficient. I sometimes say '*I'm being constructively lazy*', because I am doing some things in an easy way so that I can focus on the trickier bits. In gentle steps, a sentence describing the relationship in words is turned into a formula. This applies to the letters used for variables and the mathematical symbols themselves.

The specific form we use is important too. In many ways, using the form $F = ma$ is part of the problem. Students are rarely asked, or expected to

BOX 2 Using science connectives

I lift . . .	**Cause:** increase in height *h*
. . . a box of books away from the Earth's surface . . .	**Conditions:** mass *m*, gravitational field strength *g*
. . . so the potential energy of the box will increase . . .	**Consequence:** more energy in associated gravitational store, E_p
. . . which has to come from somewhere.	**Context:** conservation of energy, glycogen in muscles as a chemical store, ability to lift loads is limited by availability of fuels or equivalent

work out, the force applied to make an object with mass accelerate; this does not really tell a coherent story. But the maths can be made much clearer by following the narrative we have established. Just as we first describe chemical reactions using words, then word equations, then symbols, and then balanced symbols, this reduces the tendency of some students to panic when they see maths.

Our **cause** is the *force* that acts. This is acting on all of the *mass* present – a **condition** of the situation we are describing. Students may imagine the force being shared or spread out, which is an imperfect way of thinking about the inverse relationship here. As a **consequence**, there is a change in movement, which we call *acceleration*.

So in a sentence: The *force* acts on the whole *mass*, causing *acceleration*.

And in symbols: $F \div m = a$.

The link to our narrative is much clearer – and not a triangle in sight! Formula triangles are a very divisive topic for science teachers. Perhaps this is because they are often popular with students for what we see as the wrong reasons (Koenig, 2015; Southall, 2016). I prefer to avoid them where possible, because I find them a distraction from understanding the actual relationship. For some situations they can be useful for recall, but it often feels that students use them to avoid thinking, not improve it! And it is worth reminding our classes that many formulae don't fit the comfortable triangle pattern anyway, for example $E_p = mgh$.

In the case shown in Box 2, the conditions magnify or exaggerate the effect. This wording is no more ideal than the idea of a force being shared out when a mass accelerates, but that is due to the limitation of human language – and provides an excellent justification for the move to a mathematical formula! Like the stores and pathways model of energy, it is a useful stepping stone to the quantitative analysis.

An example from chemistry might be the use of ratios in titrations. Box 3 gives an example of a narrative that might lead to the mathematical description $V_2 = V_1C_1 \div C_2$.

BOX 3 A narrative example from chemistry

A more concentrated acid . . .	**Cause:** increased concentration C_2
. . . used in a titration . . .	**Conditions:** volume V_1 and concentration C_1 of alkali
. . . reaches the endpoint sooner . . .	**Consequence:** smaller volume V_2 needed
. . . which may be preferred for industrial settings.	**Context:** cost of transporting reactants/neutralising agents

In some cases the relationship may not be neatly expressed in a formula, at least not at the level our students can easily access. But breaking a narrative down into the same parts may still be helpful; among other advantages, it prompts students to consider control variables alongside the variable being considered (Box 4).

BOX 4 A narrative example from biology

More carbon dioxide is available ...	**Cause:** CO_2 percentage in air
. . . to a plant that relies on photosynthesis ...	**Conditions:** limiting factors, e.g. sunlight, temperature, water available via roots
. . . to make glucose ...	**Consequence:** amount of glucose and hence starch present
. . . which is used for a range of processes.	**Context:** plant growth and fruit production are two ready examples of processes that might increase, and are relevant to crop yield.

This might seem like a long thinking process to go through, both for us and our students. If the aim is to use and rearrange an equation with no link to the real world, this approach probably is not necessary. But mostly, an abstract understanding is not what our students need. Using this shows and reminds our students that a formula is a scientific story, one they can tell themselves.

And so they all lived happily ever after.

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